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THE DIURNAL TEMPERATURE WAVE IN A LAYERED ATMOSPHERE

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THE DIURNAL TEMPERATURE WAVE IN A LAYERED ATMOSPHERE

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Louis D. Megehee, Jr.

THE DIURNAL TEMPERATURE WAVE

IN A LAYERED ATMOSPHERE

by

Louis D. Megehee, Jr.

Lieutenant, junior grade, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
METEOROLOGY

United States Naval Postgraduate School
Monterey, California

1959

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Thesis

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IN

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from the

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ABSTRACT

The partial differential equation for heat diffusion is numerically integrated by the Runge-Kutta-Gill method. A solution is obtained for the diurnal temperature variation with a bounded coefficient of eddy diffusivity which varies periodically with time and nonlinearly with height. The surface wave is represented by the sum of a diurnal and a semidiurnal harmonic wave. The results may be interpreted to apply over a fairly broad range of diffusivity values and height. With appropriate choices of the various parameters, reasonably good agreement is obtained between theoretical and observational values of amplitude reduction and phase lag.

The author wishes to express his appreciation for the assistance and encouragement of Professors G. J. Haltiner and E. J. Stewart of the U. S. Naval Postgraduate School in this investigation.

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1. Introduction.

Of the many problems facing meteorologists, one that has received much attention is that of heat diffusion in the atmosphere. Much of this effort has been directed toward the diurnal temperature wave, and in every attempt, the concept of eddy diffusivity has been retained. The classical theory of daily temperature variations has been discussed by Sutton (7), while the more recent contributions have been summarized by Staley (6).

The Taylor heat-diffusion equation, which states that the turbulent flux of heat is proportional to the gradient of potential temperature, may be written:

$$\frac{\partial \Theta}{\partial t} = \frac{\partial}{\partial z}\left[K\frac{\partial \Theta}{\partial z}\right]. \tag{1}$$

Here t represents the time; z, the height; and Θ (z,t), the potential temperature deviation from a mean value. Generally, the coefficient of eddy diffusivity, K, is a function of both height and time in any particular location.

In many of the past studies, K has been assumed to vary, (a) linearly with height, (b) as a power of height, and (c) as a bounded exponential function of height. In each of these cases, the assumed functional form of K was assumed to hold throughout the atmosphere. Recently, however, de Vries (1) has studied:

> the case of an atmosphere consisting of several layers, in each of which K is expressed as a different function of height.

For an example, he proposed three layers; a laminar sublayer, a turbulent boundry layer, and a "free" atmospheric layer. In general, each of these layers, and in particular, the turbulent boundary layer, can be further subdivided into layers. He also gave

analytical solutions for several particular cases; (a) K = constant, (b) $K = \alpha(z+z_0)^{\beta}$, and (c) $K = a[1 - b \exp(-cz)]$. Case (a) yields a solution in terms of a rather complicated exponential function, (b), a solution in terms of Bessel functions of the first kind, and (c), a solution in terms of a hypergeometric function. However, no numerical values were given, and the results are not suitable for practical purposes.

It is the purpose of this investigation to present a numerical solution of Eqn. (1), with appropriate boundary conditions, for a particular case of an atmosphere composed of an arbitrary number of layers.

2. The Heat Diffusivity Coefficient.

From physical considerations, K, the heat diffusivity, may be expected to increase in the lower layers, but should not continue to increase indefinitely with height. Hence, a bounded diffusivity coefficient, which increases with height from the surface to some fixed level and then decreases with height to some residual value at higher levels, would seem more suitable than either the case of K increasing linearly with height or of K increasing as a simple power of height. Upon examination of Fig. (1), which is based on observations taken at the micrometeorological tower at the University of Washington (2), (3) and Mildner's pilot balloon observations taken near Leipzig (5), it is seen that no single functional form of K will suffice. The values of K from the University of Washington data were obtained by methods based on energy continuity at the earth's surface, while those from the Leipzig data were actually values of eddy viscosity. These values of eddy viscosity were used as an approximation to eddy diffusivity, since they showed the desired variation with height, and since values of eddy diffusivity at sufficiently great heights are difficult to obtain. Upon dividing the atmosphere into four layers, and then fitting functions of height to each layer, the following form of K resulted: [see Fig. (2)]

$$K = f(z) = \begin{cases} a_1 z^2, & 0m \le z < 50m \\ a_2 z + a_3, & 50m \le z < 228m \\ a_4 \exp[a_5(z-h)], & 228m < z < 460m \\ a_6, & 460m \le z. \end{cases} \qquad (2)$$

The quantities $a_1, a_2, a_3, a_4, a_5, a_6$, and h are constants, appropriate values of which will be assigned later. The discontinuity which

Figure 1

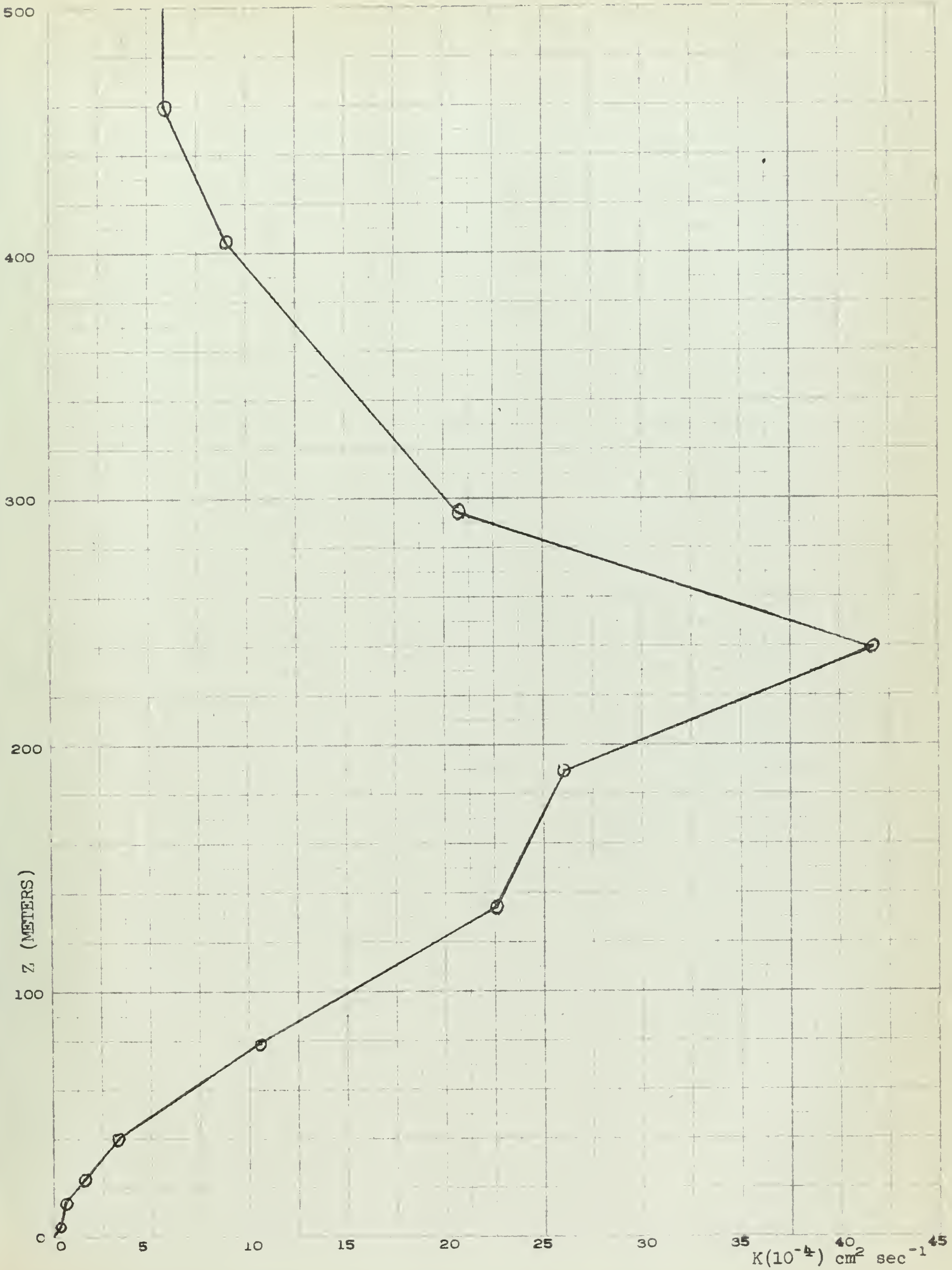
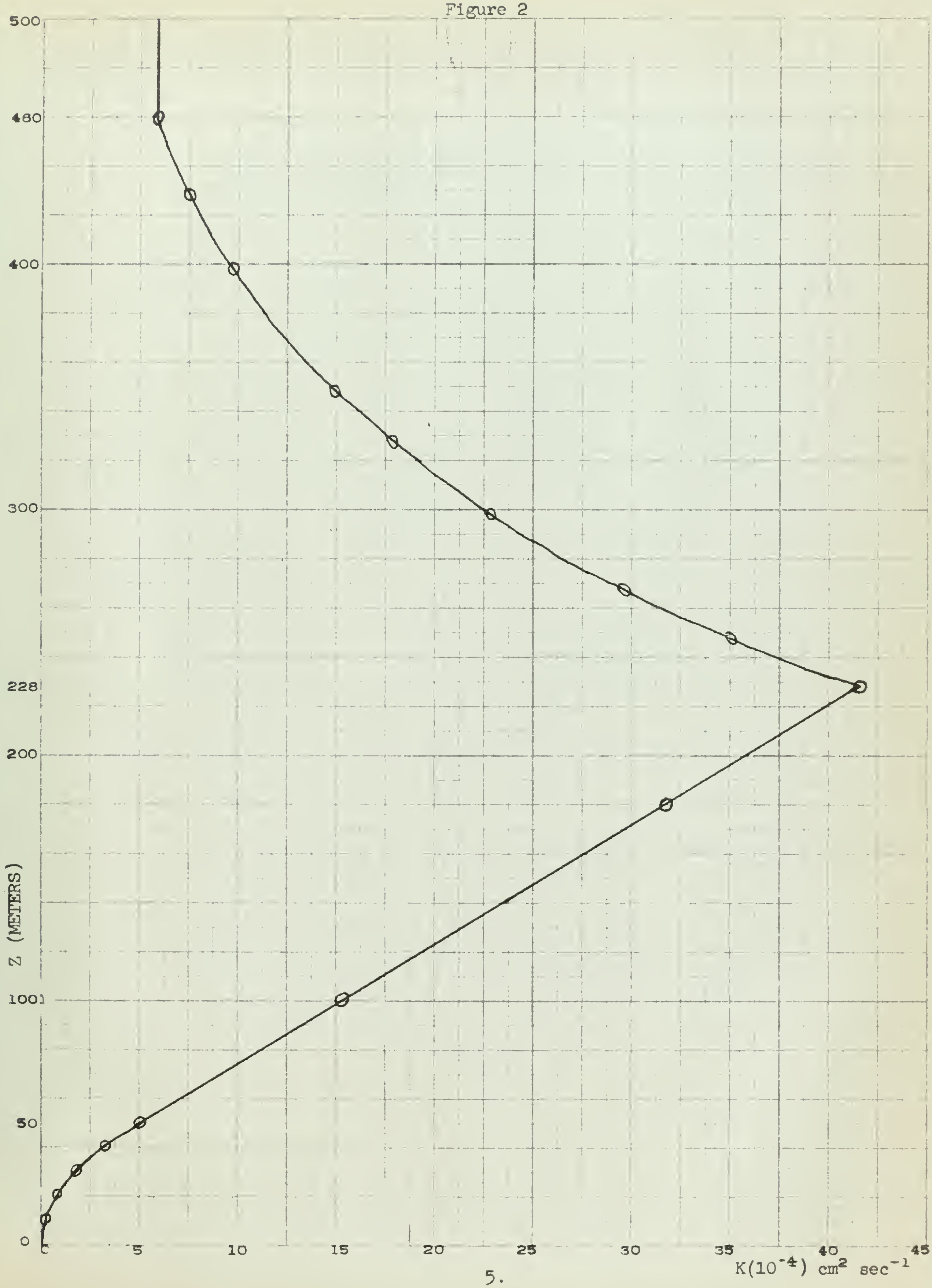

500
400
300
200
100
0
Z (METERS)
0
5
10
15
20
25
30
35
40
45
K(10^-4) cm^2 sec^-1

Figure 2


500
460
400
300
228
200
100
50
0
Z (METERS)
0
5
10
15
20
25
30
35
40
45
K(10^{-4}) cm^2 sec^{-1}

occurs at $z = 228$ meters is resolved by defining $\frac{\partial K}{\partial z}$ at this level as zero.

Haltiner (4) suggests that K should have a diurnal variation, and that

> it appears reasonable to expect that K would more or less follow the surface temperature wave, reaching maximum and minimum values near the time when the temperature is maximum and minimum, respectively.

The eddy diffusivity, K, can now be expressed as a function of height times a function of time, i. e., $K = f(z)g(t)$, where $g(t)$ is defined:

$$g(t) = 1 + b(\sin\omega t + c \sin 2\omega t) \qquad (3)$$

Here b and c are constants, and $\omega = 2\pi/86{,}400 \text{ sec}^{-1}$.

Appropriate boundry conditions associated with Eqn. (1) will now be chosen. These are frequently taken to be:

$$\theta = 0 \text{ for } z = \infty, \text{ and all } t; \qquad (4)$$

$$\theta = \theta_0(t) \text{ for } z = 0; \qquad (5)$$

where $\theta_0(t)$ is defined by:

$$\theta_0(t) = d(\sin\omega t + c \sin 2\omega t) \qquad (6)$$

Here d is a constant and c and ω are the same as in Eqn. (3).

3. Transformation of Coordinates.

In order to place the basic equations into a form which will give a broader interpretation of the results, and to scale these equations for computational purposes, let:

$$z = 100q\,\sigma\ , \text{ and } t = 3600\tau\ . \tag{7}$$

Here q is a constant, while τ and σ are the new variables. When t is in seconds, the units of τ are hours, and when z is in centimeters, the units of σ are in "q-meters"; i. e., in 1-meter units when q = 1, 2-meter units when q = 2, etc. With this transformation, (7), Eqs. (1) and (2) become:

$$\frac{\partial \theta}{\partial \tau} = \frac{0.36}{q^2}\frac{\partial}{\partial \sigma}\left[K\frac{\partial \theta}{\partial \sigma}\right], \tag{8}$$

$$K = F(\sigma)\cdot g(\tau) = g(\tau)\begin{cases} a_1'\sigma^2 \\ a_2'\sigma + a_3 \\ a_4 \exp[-a_5'(\sigma - h')] \\ a_6 \end{cases} \tag{9}$$

Here ω must now be taken as $2\pi/24$, and;

$a_1' = a_1(100q)^2$, $\qquad a_2' = a_2(100q)$,

$a_5' = a_5(100q)$, $\qquad h' = h/100q$.

The boundary conditions, Eqs. (4) and (5), remain identical in form.

Both observation and theory indicate that the most pronounced variations of temperature occur near the surface of the earth. Therefore, a small grid distance is desirable at low levels, while at higher levels, a small grid distance is not necessarily needed. This suggests that the following transformation of the vertical

coordinate may be useful:

$$s = \ln \sigma . \tag{10}$$

With this transformation, the lower boundary condition, Eqn. (5), may apply at a height of q meters corrosponding to $\sigma = 1$, however, to reduce surface effects, it is convenient to select $\sigma = 2$. Actually the vertical coordinate system is arbitrary, and can be chosen to have its base at any level. By utilizing Eqn. (10), Eqs. (8) and (9) now become:

$$\frac{\partial \theta}{\partial t} = \frac{0.36\, a_7}{g^2} e^{-2s}\left[K\frac{\partial^2\theta}{\partial s^2} + \left(\frac{\partial K}{\partial s} - K\right)\frac{\partial \theta}{\partial s}\right], \tag{11}$$

$$K = F(s)\,g(t) = a_7 g(t) \begin{cases} a_1' e^{2s} \\ a_2' e^{s} + a_3 \\ a_4 \exp\left[-a_5'(e^{s} - h')\right] \\ a_6 \end{cases} \tag{12}$$

Here a_7 is taken as 10^6, and the other constants, a_1', a_2', a_3, a_4, and a_6, must be scaled by a factor of 10^{-6}.

4. Finite Difference Equations.

In order to obtain a numerical solution, the derivatives of Θ with respect to s and τ in Eqn. (11) must be replaced by appropriate finite difference forms. The problem is then reduced to that of solving a system of linear algebraic equations in the values of Θ over a grid of points covering the desired range. Expressing $\frac{\partial^2 \Theta}{\partial s^2}$ and $\frac{\partial \Theta}{\partial s}$ in Eqn. (11) as finite difference ratios, we obtain:

$$\frac{d\Theta_i}{d\tau} = \frac{0.36\, a_7}{g^2} e^{-2s} \left[(K_i)\left(\frac{\Theta_{i+1} - 2\Theta_i + \Theta_{i-1}}{\ell^2} \right) + \left\{ \left(\frac{dK}{ds}\right)_i - K_i \right\} \left(\frac{\Theta_{i+1} - \Theta_{i-1}}{2\ell} \right) \right]$$

$$i = 1, 2, 3, ----, n. \qquad (13)$$

Here the subscript i designates the i'th level of the vertical grid at height i ℓ, where ℓ is the vertical distance between grid points in units of s, that is, a pure number, since s is dimensionless.

The partial differential equation, Eqn. (1), has now been reduced to a system of ordinary differential equations which will be integrated numerically by the Runge-Kutta-Gill method.

5. The Constants.

Next, appropriate values of ℓ, n, and Δt, as well as the constants a_1', a_2', etc., will now be chosen. Since the vertical coordinate is $s = \ln \sigma$, $\ell = \Delta s$, a constant. With a choice of $\ell = \ln 2$, the i'th equation of the system defined by Eqn. (13) applies at the height 2^i q-meters. Therefore, the successive values of i, beginning with the lower boundary condition, apply to the levels, 2, 4, 8, 16, 32, 64, etc. q-meters. To reduce the integration time, yet still give a fairly detailed picture of the vertical structure, the upper boundary condition, Eqn. (4), is chosen to apply at the height of 1024 q-meters. For this choice, $\Theta_9 \equiv 0$; and the system, Eqn (13), to be solved consists of eight simultaneous ordinary differential equations, together with Eqn. (6) for Θ_0. A numerical solution for this system was then obtained by the Runge-Kutta-Gill method on a National Cash Register 102A electronic computer. The choice of an appropriate time interval depended intimately upon the size of the eddy diffusivity. As the latter increased, Δt had to be decreased in order to maintain computational stability and reasonable accuracy.

The following numerical values of the other constants were chosen as being representative:

$$
\begin{aligned}
a_1' &= 0.02\ (\mathrm{sec}^{-1}), & a_2' &= 2050\ (\mathrm{cm/sec}), \\
a_3 &= -52{,}500\ (\mathrm{cm^2/sec}), & a_4 &= 415{,}000\ (\mathrm{cm^2/sec}), \\
a_5' &= 0.0085, & a_6 &= 58{,}000\ (\mathrm{cm^2/sec}), \\
b &= 0.3, & c &= 0.3, \\
d &= 0.05\ (^{\circ}\mathrm{C}), & h' &= 228.
\end{aligned}
\tag{14}
$$

The choice of a_1', a_5', a_3, a_4, and a_6 gives a 328-fold increase of K with height from the surface to 228 q-meters, and a 14-fold decrease of K with height above 228 q-meters. Also, the choice of

b and c gives rise to a 21-fold diurnal variation of K with time. The value of d was chosen because it keeps the magnitude of $\theta_0 < 0.1$.

With the value of $a_7 = 10^6$, the integration interval was prohibitively short for the computer used; therefore a smaller value of $a_7 = 6.25 \times 10^4$ was used. This permitted a time interval of 1/30 hour.

The value of $a_7 = 6.25 \times 10^4$ gives the following range of the coefficient of eddy diffusivity, Eqn. (12), in units of cm^2/sec:

at surface,	$K_{MAX} = 0.15 \times 10^2 q^2$,	$K_{MIN} = 0.714\ q^2$,
at 20 q-m,	$K_{MAX} = 0.15 \times 10^3 q^2$,	$K_{MIN} = 7.14\ q^2$,
at 228 q-m,	$K_{MAX} = 0.492 \times 10^4 q^2$,	$K_{MIN} = 230\ q^2$,
at 460 q-m,	$K_{MAX} = 0.35 \times 10^3 q^2$,	$K_{MIN} = 17\ q^2$.

For $q \leq 1$, the values of K are somewhat small for typical atmospheric conditions, especially in the lowest layers. This is reflected in the results by a large amplitude reduction and phase lag with increasing height. However, such values of K are not unrealistic for winter.

6. Results.

The results of the integration are given by Fig. (3) and Table (1), Fig. (3) being the graphical interpretation of Table (1). In Fig. (3), amplitude reduction and phase lag, in hours, are plotted against height, in q-meters. As can be seen from Fig. (3), the amplitude reduction and phase lag are quite large, especially when compared with observed values. (See Fig. (4) which is based on observations taken at the micrometeorological tower at the University of Washington). The large amplitude reduction and phase lag from the theoretical results, Fig. (3), **are due largely** to the reduction of a from 10^6 to 6.25 x 10^4. However, by proper selection of the value of q, fair agreement between theoretical and observed values can be obtained. Fig. (3) also shows the amplitude reduction and phase lag for the minimum at three levels. From these three levels, it can be seen that the amplitude reduction is less for the maximum than for the minimum, the latter corresponding to the lower nocturnal diffusivity. This difference is appreciable in that at the eight q-meter level, the maximum is about 43°/o of the surface value of θ while the minimum is only 19°/o of the surface minimum. At lower levels, however, this difference is much smaller. The phase lag increases with height for both the maximum and the minimum, however, the lag of the maximum is greater than that of the minimum at any particular level. This does not seem consistent with greater amplitude reduction of minima compared to maxima. Possibly, these differences would vary in succeeding maxima and minima, as greater time elapses from the initial conditions.

By a proper choice of q, some comparisons between observed and theoretical results may be made. Table (2) shows amplitude reduction and phase lag from observations taken at Leafield, along with

Figure 3

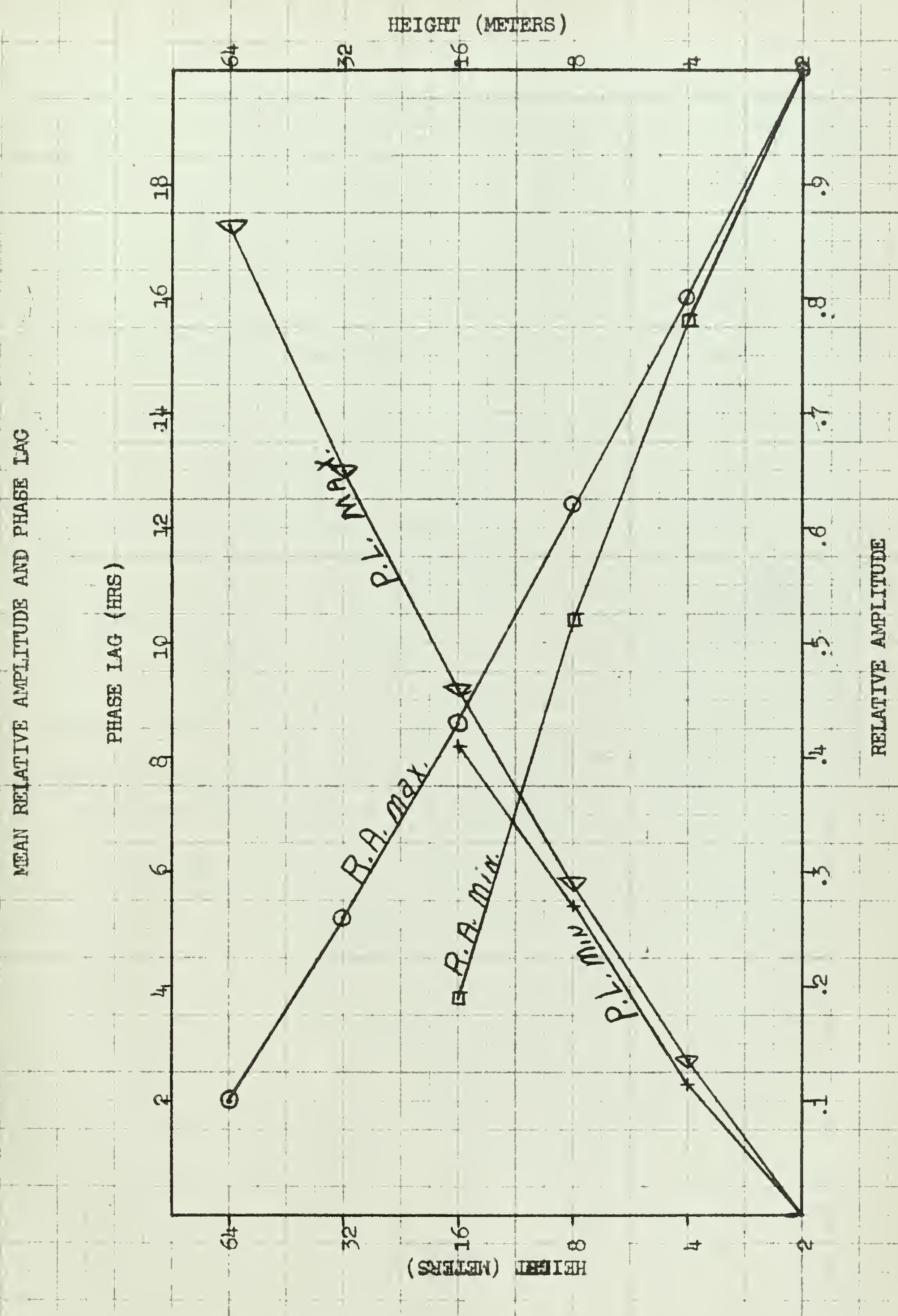

MEAN RELATIVE AMPLITUDE AND PHASE LAG
PHASE LAG (HRS)
HEIGHT (METERS)
RELATIVE AMPLITUDE
HEIGHT (METERS)
R.A. max.
R.A. min.
P.L. MAX.
P.L. MIN
2
4
6
8
10
12
14
16
18
.1
.2
.3
.4
.5
.6
.7
.8
.9
64
32
16
8
4
2

TABLE I

MEAN RELATIVE AMPLITUDE AND PHASE COMPUTATIONS

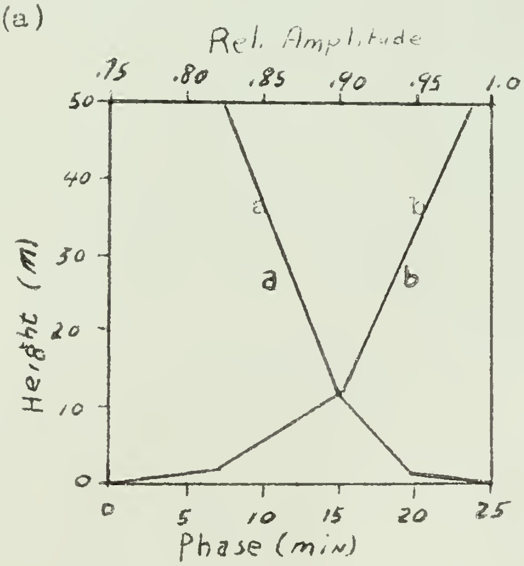
Level	Height m	Time (hrs.) max	Time (hrs.) min	Amplitude maximum	Amplitude minimum	Rel. Amp. MAX	Rel. Amp. Min.	Phase Lag MAX	Phase Lag min.
0	2	4.4	19.6	.056806	.056818	1	1	0	0
1	4	7.1	21.9	.045445	.044318	.80	.78	2.7	2.3
2	8	10.2	25.0	.035220	.029546	.62	.52	5.8	5.4
3	16	13.6	27.8	.024427	.011795	.43	.19	9.2	8.2
4	32	17.4	—	.014770	—	.26	—	13.0	—
5	64	21.7	—	.005681	—	.10	—	17.3	—
6	128	33.0	—	.000086	—	.0015	—	28.6	—

Figure 4

MEAN R. A. AND P. I. OBSERVED AT UNIV. OF WASH.

MICROMETEOROLOGICAL TOWER FOR 23 DAYS OF AUG. 1950

R. A. (a) P. I. (b)



theoretical values for $q = 10$ and $q = 20$. For both values of q, there is fair agreement with observed values, however, both the theoretical amplitude and phase lag increase with height much more rapidly than do the observed values. Table (3) compares theory with observed data at the Eiffel Tower. Here q is taken to be 60 and 33, the value of 60 comparing to the summer data and the 33, to the winter data. Table (4) shows some observations taken at the University of Washington. By the choice of $q = 8$, the amplitude compares fairly well; but where the observed phase lags are the order of minutes, those from theory are the order of hours. In Table (5), the data from Porton is compared to theory. For the choice of $q = 2.5$, the comparison is fair, but once again, shows the theoretical phase lag and amplitude reduction much greater than the observed. Table (6) compares the amplitude reduction in summer and winter at Leafield with theory. Here $q = 5$ was chosen for winter, and $q = 12$, for summer. For the winter case the theoretical and observed values compare very well, but for summer, the theoretical amplitude reduction is significantly greater than the observed value.

Table.2.

Amplitude reduction (A. R.) and phase lag (P. L.) taken for the cases

q = 10 and q = 20 versus observations taken at Leafield, England.

Theory				Height	Observation	
q = 10		q = 20				
A. R.	P.L.(hrs)	A. R.	P. L.(hrs)	(meters)	A. R.	P.L.(hrs)
.95	.83	1.0	0.0	25	.90	—
.90	1.5	1.0	0.0	30	—	1.2
.76	3.7	.95	.83	50	.84	—
.72	4.5	.90	1.5	60	—	1.5
.60	6.2	.78	3.3	90	—	1.66
.58	6.5	.77	3.7	100	.77	—
.53	7.2	.72	4.5	120	—	—

Table.3.

Amplitude reduction from the Eiffel Tower data versus theoretical

values for q = 60 and q = 33.

Theory A. R.		Height	Observation A. R.	
q = 60	q = 33	(meters)	Summer	Winter
.88	.77	195	.88	.83
.76	.60	300	.75	.60

Table.4.

Amplitude reduction and phase lag for the case of q = 8 versus observations taken at the University of Washington.

Theory		Height	Observation	
A. R.	P. L.	(Meters)	A. R.	P. L.
1.0	0.0 hrs.	15	.91	16 min.
.82	2.4 "	30	.86	18 "
.75	3.9 "	45	.83	23 "

Table.5.

Amplitude reduction and phase lag for the case of q = 2.5 versus observations taken at Porton, England.

Theory		Height	Observation	
A. R.	P. L.	(Meters)	A. R.	P. L.
.88	1.1 hrs.	7	.73	.95 hrs.
.66	4.8 "	17	.65	1.2 "

Table.6.

Amplitude reduction from the Leafield data versus theoretical values for q = 5 and q = 12.

Height in Meters	10	25	50	75	100
Leafield (a) June	1.0	.90	.84	.79	.77
A.R. (b) December	1.0	.74	.54	.44	.40
Theory (c) q = 12	1.0	1.0	.85	.75	.67
A.R. (d) q = 5	1.0	.76	.58	.45	.39

7. Conclusions.

A numerical solution has been presented for the diurnal temperature variation with a coefficient of eddy diffusivity which is a function of height and time. By suitable selection of several parameters, reasonably good agreement has been obtained between theory and observation.

Improvement in the results may possibly be achieved by improving the functional form of the diffusivity coefficient, especially in the lowest layers. It would also be desirable to include such parameters as surface roughness, stability, wind velocity, etc. With a diffusion coefficient in terms of these parameters, as well as height and time, a solution for the diurnal temperature wave would have a much broader application.

8. Bibliography.

(1) de Vries, D. A., 1957: On the integration of the heat-conduction equation with periodic variation of temperature. J. Meteor., 14, 71-76.

(2) Fleagle, R. G., and F. I. Badgley, 1951: Atmospheric turbulence study. (Occ. Rep. 1, Contract AT(45-1)-236), Seattle, Univ. of Wash., 26 pp.

(3) _____, 1952: Atmospheric turbulence study. (Occ. Rep 1, Contract AT(45-1)-236), Seattle, Univ. of Wash., 47 pp.

(4) Haltiner, G. J., 1958: The diurnal temperature wave with a coefficient of diffusivity which varies periodically with time and exponentially with height. J. Meteor., 15, 317-323.

(5) Mildner, P., 1932: Uber die Turbulenz des Windes beobachtet mit Hilfe von doppelt und einfach visierten Pilotballonen. Beitr. Phys. Atm., 19, 151.

(6) Staley, D. O., 1956: The diurnal temperature wave for bounded eddy conductivity. J. Meteor., 13, 13-20.

(7) Sutton, C. G., 1953: Micrometeorology. New York, McGraw-Hill, 333 pp.

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